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Geometric Amplitude Formulation of Measurement in Hyperhamiltonian Quantum Mechanics

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Abstract

We propose a geometric reformulation of measurement in Hyperhamiltonian Quantum Mechanics (HHQM), replacing the inverse-distance propensity by a complex transition amplitude defined on the state manifold. The amplitude depends on geodesic distance and a phase functional associated with the hyperquantum bundle, leading to a normalized Born-type probability rule. In a minimal bipartite setting, we show that interference between a small number of transition channels can produce joint probability distributions with nonclassical correlations. In particular, a simple twochannel model yields a cosine correlator and attains the Tsirelson bound while preserving no-signaling.

The resulting structure suggests that Bell-type correlations in HHQM, if realized, arise from interference of geometric transition amplitudes rather than from spacetime dynamics. The explicit kernel constructed here should be regarded as an effective realization within this framework; a full derivation from the underlying bundle geometry remains an open problem.

1. Introduction

Hyperhamiltonian Quantum Mechanics (HHQM), originally introduced in [37], is a geometric modification [23], [3], [11], [8] of the standard Quaternionic Quantum Mechanics [1], in which quantum states are quaternionic rays and also the fibers (*possible worlds*) of a hyperkähler bundle [5] (*monocosm* of the system).

The measurement mechanism of HHQM is based on the notion of propensity, developed by Karl Popper and others [31] , [25], [15], [26]. Here we utilize a specic technical version of propensity as inverse geodesic distance between possible worlds. Although independent of the quantum probability framework conceptually, propensity is mostly unfamiliar to the working physicist and not suciently developed to cover the full range of quantum phenomena like interference effects, which play a central role in standard quantum theory [42]. In this paper we construct a minimal bridge between the propensity mechanism and the standard quantum probability-based measurement theory.

The paper is organized as follows. In Sec. 2 we give a brief outline of HHQM. In Sec. 3 we introduce the amplitude-based reformulation of HHQM measurement and the associated probability rule.

In Sec. 4 we analyze bipartite systems and exhibit a minimal Bell-type kernel. In Sec. 5 we provide a geometric interpretation in terms of holonomy and discuss the status and limitations of the construction.

We conclude in Sec. 6 with a summary and outlook.

2. HHQM basics

2.1 Kinematics

Each quaternionic ray (the set of nonzero quaternions) has the following properties: (a) it is a locally compact Lie group, under quaternion multiplication; (b) it is automatically endowed with a bi-invariant Haar measure generated by the group multiplication;

(c) it is a smooth real four dimensional manifold; (d) it has a natural closed (FLRW) metric induced by quaternion algebra operations [36]; (e) it has a Lebesgue measure induced by the FLRW metric; (f) it visually looks like an hourglass (embedded in a higher-dimensional Minkowski space), flaring toward Past and Future. The standard Complex Quantum Mechanics with antihermitian operators is a degenerate case of HHQM: two out of four dimensions are collapsed in each possible world, destroying all the structures described above.

Remark 2.1. Two-measure structure [20] on spacetime has a natural interpretation: while the standard Lebesque measure takes care for matter/radiation density, the Haar measure, as purely spacetime, is assumed responsible for vacuum energy density. Indeed, all 3-sphere slices of the hourglass are produced by translations of the unit "waist" 3-sphere along left and right invariant vector fields. Since this measure is invariant, all 3-sphere slices have the same (constant) volume, while their Lebesgue volume increases. This means that matter density decreases toward Past and Future, and vacuum energy density, being constant, catches up and eventually overtakes matter/radiation density, producing accelerated expansion without an explicit cosmological constant. We discuss it in detail elsewhere, but this almost trivial explanation for the dark energy phenomenon demonstrates the usefulness of the HHQM framework [39].

2.2 Semantics

The interpretation of HHQM is Observer-centric: the universe is considered a totality of the Observer's elementary experiences (observations) structured via his logic [35], [38]. For any physical system its quantum states are copies (possible worlds) of the closed FLRW spacetime (a *cosmology*) in the GR sense, with the scale factor as a free parameter. It is important to stress that the only spacetime metric used in HHQM is closed FLRW. However the reason for its appearance originates from the type of logic of the Observer [35], rather than GR. HHQM uses quaternionic antihermitian operators, with quadruples of real numbers as eigenvalues, and quaternionic regular maps, as a part of its measurement apparatus.

2.3 Dynamics

In this approach, dynamics is generated by a hyperhamiltonian (hypersymplectic) ow [16] [17] in the bundle superposition of three hamiltonian flows, with the *propesity metric* as the main dynamical variable. Since only closed FLRW metric (smooth and nondegenerate everywhere) is used in every physical conguration, HHQM is singularity-free kinematically, though singularity-like issues (caustics, bifurcations, etc.) can be encountered during system evolution. Since the dynamics is extremely rich due to complexity of the transverse geometry of the bundle, HHQM neither uses nor needs full EFEs. As mentioned earlier, propensity metric (the Riemannian part of the hypersymplectic structure) has a natural meaning in HHQM: it measures geodesic distance/accessibility between possible worlds.

3. From Propensity to Amplitude: A Born-like Geometric Measure-ment Rule for HHQM

The central idea is to replace the inverse-distance propensity with an amplitude whose modulus is governed by geodesic distance, while its phase is associated with geometric data [23] of the hyperquantum bundle. This leads to a normalized Born-type probability rule [10] and introduces the possibility of interference between distinct transition channels.

Within this amplitude-based formulation [41], [14], [21], [27], [33], [12] [24], [12], we consider a simple bipartite setting and show that interference between a small number of transition channels can give rise to joint probability distributions with nonclassical correlations. In particular, we construct a minimal twochannel model in which the resulting correlator takes a cosine form and attains the Tsirelson bound [40] while preserving no-signaling.

The construction provides a natural geometric interpretation in terms of holonomy of the underlying bundle connection, suggesting that such correlations, if realized within HHQM, originate from state-space geometry rather than from nonlocal spacetime dynamics.

Kinematic nonlocality [19] in HHQM arises from the group structure on spacetime which associates to any two events *a* and *b* a third even *ab*, independently of spatial separation [30].

Let $(\mathcal{X}, g)$ denote the state bundle, with geodesic distance $d(\phi, \psi)$. We replace the inverse-distance

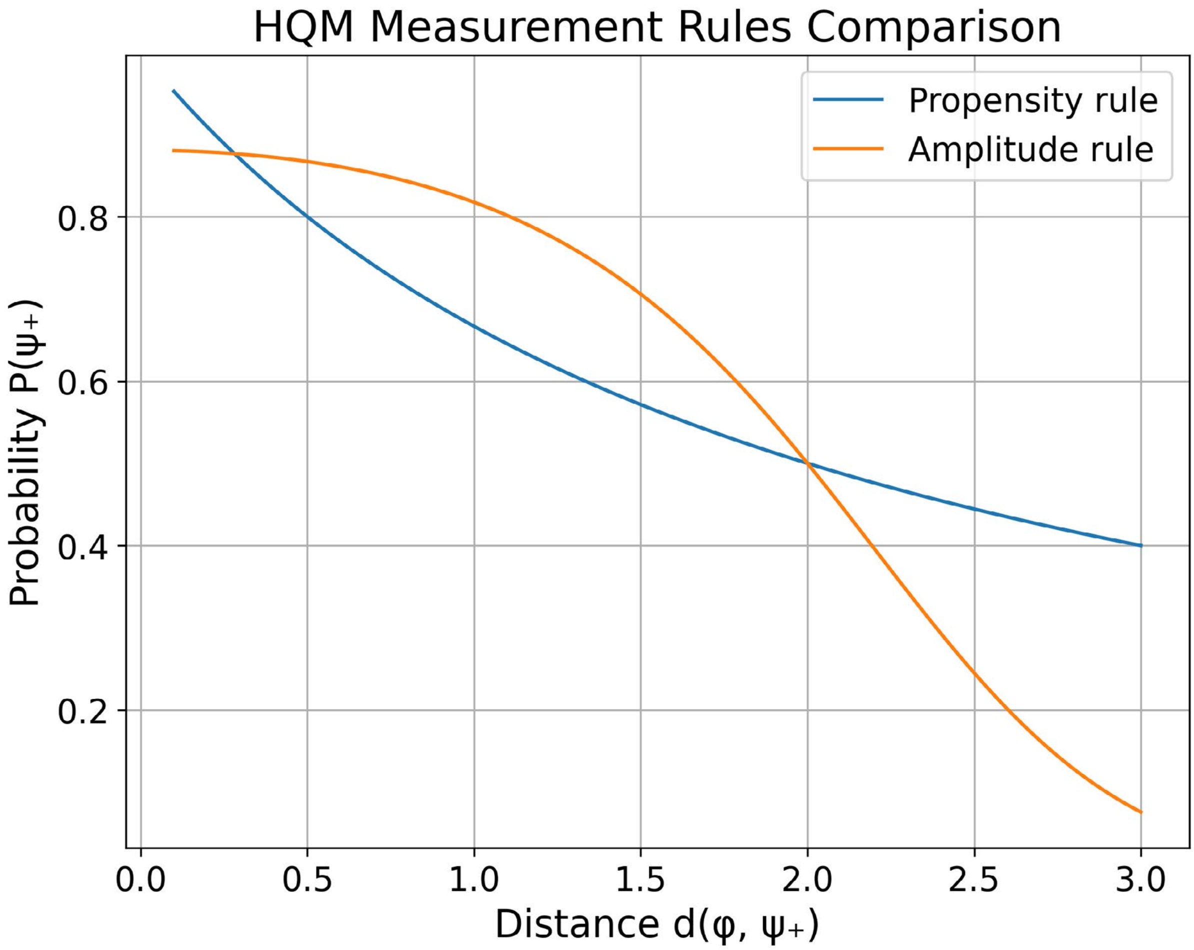

Figure 1: Comparison of HHQM measurement rules. The normalized inverse-distance propensity rule suppresses distant outcomes linearly, whereas the amplitude-based Gaussian rule suppresses them exponentially and introduces a natural measurement-scale parameter.

propensity

$$\rho(\phi,\psi)=\frac{1}{d(\phi,\psi)} \tag{1}$$

by the transition amplitude

$$A_M(\phi,\psi)=\Delta(\phi,\psi)^{1/2}\exp\left[-\frac{d(\phi,\psi)^2}{4\tau_M}\right]\exp(i\,\mathcal{S}_M(\phi,\psi))\,. \tag{2}$$

The corresponding Born-type rule is [6]

$$P(\psi_i\mid\phi;M)=\frac{|A_M(\phi,\psi_i)|^2}{\sum_j|A_M(\phi,\psi_j)|^2}. \tag{3}$$

4. Bipartite HHQM Toy Model and Bell-Type Structure

We consider the bipartite joint probability distribution derived from the geometric transition amplitude in HHQM [7], [13], [4], [30]. For binary outcomes $i,j\in\{+1,-1\}$ for subsystems A and B, and local measurement settings $\alpha,\beta\in[0,2\pi)$, the joint amplitude is postulated as

$$A_{ij}(\alpha,\beta)=\frac{1}{2\sqrt{2}}\Big(1+ij\,e^{i(\alpha-\beta)}\Big).$$

The corresponding joint probability is

$$P_{ij}(\alpha,\beta)=|A_{ij}(\alpha,\beta)|^2=\frac{1}{4}\Big(1+ij\cos(\alpha-\beta)\Big).$$

Step 1: Correlation function
The correlation function is

$$E(\alpha,\beta)=\sum_{i,j=\pm1} ij\,P_{ij}(\alpha,\beta).$$

Substituting P_{ij} :

$$E(\alpha,\beta)=\sum_{i,j} ij\cdot\frac{1}{4}\Big(1+ij\cos(\alpha-\beta)\Big)=\frac{1}{4}\left[\underbrace{\sum_{i,j} ij}_{=0}+\cos(\alpha-\beta)\underbrace{\sum_{i,j}(ij)^2}_{=4}\right].$$

Since $(ij)^2 = 1$ for all i, j, we obtain

$$E(\alpha,\beta)=\cos(\alpha-\beta).$$

Step 2: CHSH expression
The CHSH parameter is defined as

$$S=E(\alpha,\beta)-E(\alpha,\beta')+E(\alpha',\beta)+E(\alpha',\beta').$$

For a local hidden variable theory, $|S| \le 2$ (CHSH inequality). Quantum mechanics allows up to $|S|\le 2\sqrt{2}$ (Tsirelson bound).

Step 3: Choice of settings
To achieve the maximal violation with $E(\alpha, \beta) = \cos(\alpha - \beta)$, we use the standard optimal angles:

$$\alpha=0,\quad \alpha'=\frac{\pi}{2},\quad \beta=\frac{\pi}{4},\quad \beta'=\frac{3\pi}{4}.$$

Step 4: Compute each term

$$\begin{aligned}
E(\alpha,\beta)&=\cos\big(0-\tfrac{\pi}{4}\big)=\cos\big(-\tfrac{\pi}{4}\big)=\tfrac{\sqrt{2}}{2},\\
E(\alpha,\beta')&=\cos\big(0-\tfrac{3\pi}{4}\big)=\cos\big(-\tfrac{3\pi}{4}\big)=\cos\big(\tfrac{3\pi}{4}\big)=-\tfrac{\sqrt{2}}{2},\\
E(\alpha',\beta)&=\cos\big(\tfrac{\pi}{2}-\tfrac{\pi}{4}\big)=\cos\big(\tfrac{\pi}{4}\big)=\tfrac{\sqrt{2}}{2},\\
E(\alpha',\beta')&=\cos\big(\tfrac{\pi}{2}-\tfrac{3\pi}{4}\big)=\cos\big(-\tfrac{\pi}{4}\big)=\tfrac{\sqrt{2}}{2}.
\end{aligned}$$

Step 5: Summation

$$\begin{aligned}
S&=\Big(\tfrac{\sqrt{2}}{2}\Big)-\Big(-\tfrac{\sqrt{2}}{2}\Big)+\Big(\tfrac{\sqrt{2}}{2}\Big)+\Big(\tfrac{\sqrt{2}}{2}\Big)\\
&=\tfrac{\sqrt{2}}{2}+\tfrac{\sqrt{2}}{2}+\tfrac{\sqrt{2}}{2}+\tfrac{\sqrt{2}}{2}\\
&=4\cdot\tfrac{\sqrt{2}}{2}=2\sqrt{2}.
\end{aligned}$$

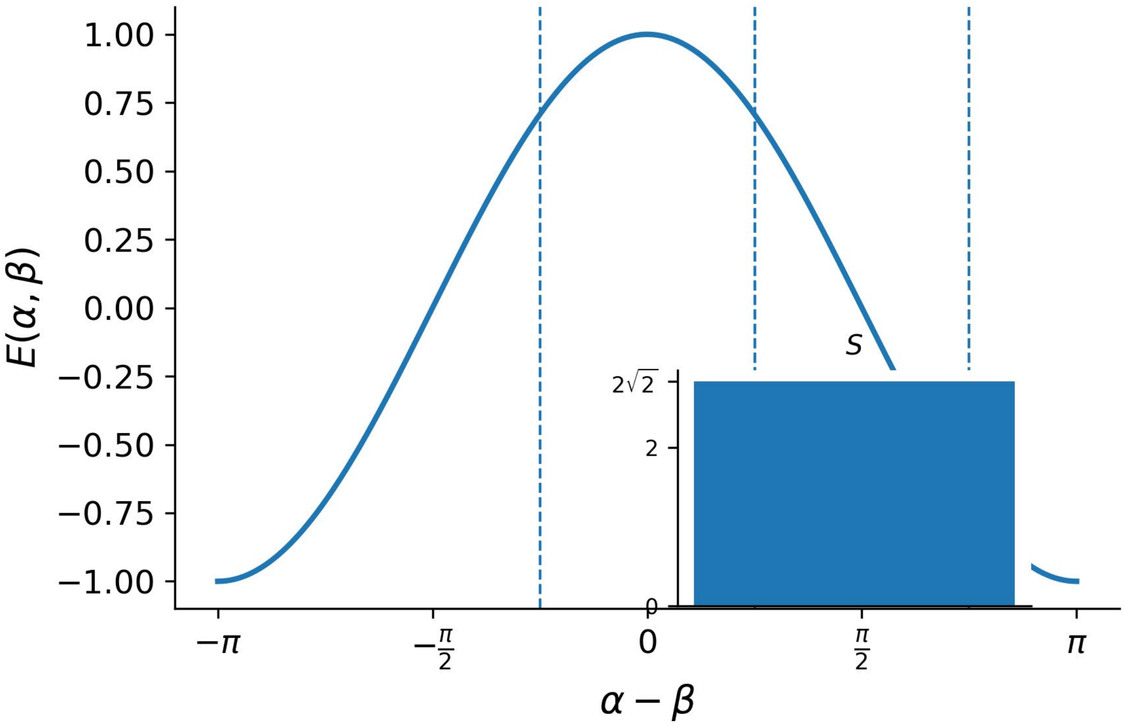

Figure 2: HHQM Bell correlator $E(\alpha, \beta) = \cos(\alpha - \beta)$ arising from interference of two geometric transition channels. Dashed lines indicate the angle differences corresponding to the CHSH settings. The inset shows the resulting CHSH parameter $S = 2\sqrt{2}$, exceeding the classical bound $S \leq 2$ while preserving no-signaling.

Step 6: Conclusion
Thus the HHQM amplitude model achieves the Tsirelson bound:

$$|S| = 2\sqrt{2}.$$

5. Geometric Interpretation of the Cosine Correlator via Holonomy

In HHQM framework, the state space (monocosm) is a hyperkähler manifold $\mathcal{M}$ (the total space of the hyperquantum bundle). The base space is the quaternionic projective space, and each fiber is isomorphic to the Lie group $\mathcal{H}^\circ = \mathbb{H} \setminus \{0\}$, which carries a natural closed FLRW metric and a hyperkähler structure [2], [34], [43], [22], [32].

Step 1: Geometric transition amplitude
Let $\phi, \psi \in \mathcal{M}$ be two states (points on the bundle). define the geometric transition amplitude

$$\mathcal{A}(\phi \to \psi) = \Delta(\phi, \psi)^{1/2} \exp\left(-\frac{d(\phi, \psi)^2}{4\tau_M}\right) \exp(i\,\Theta_{\mathrm{hol}}(\phi, \psi)),$$

where:

- $d(\phi, \psi)$ is the geodesic distance on $\mathcal{M}$ with respect to the propensity metric $\mathbf{g}$.
- $\tau_M > 0$ is a measurement scale parameter (smoothing length).
- $\Delta(\phi, \psi)$ is a Van Vleck–Morette determinant ensuring unitarity at short distances.
- $\Theta_{\mathrm{hol}}(\phi, \psi)$ is the holonomy phase of the hyperkähler connection along a reference geodesic.

Step 2: Holonomy phase from the hyperkähler bundle
The hyperquantum bundle $\pi : \mathcal{M} \to \mathcal{PV}_{\mathbb{H}}$ has a natural connection ω (the fundamental connection). For a path γ from ϕ to ψ, the holonomy is an element of the structure group $\mathcal{H}^\circ$. Since the ber is abelian in its center $U(1)$, we can extract a $U(1)$ phase:

$$\Theta_{\mathrm{hol}}(\phi, \psi) = \oint_\gamma \omega \pmod{2\pi}.$$

In a local trivialization over the base, this phase decomposes into:
1. A **dynamic phase** proportional to the geodesic length (already absorbed in the Gaussian factor in a more refined treatment, but here kept separate for clarity).
2. A **geometric phase** [9], [28] (PancharatnamBerry type) depending only on the base ray and the measurement settings.

For a bipartite system with settings α (for subsystem A) and β (for subsystem B), the holonomy phase difference between two distinct transition channels can be modeled as:

$$\Delta\Theta_{ij}(\alpha,\beta) = (\alpha-\beta) + \pi\,\frac{1-ij}{2}, \qquad i,j\in\{+1,-1\}.$$

This expression combines:
- A continuous setting-dependent part $(\alpha-\beta)$ arising from the $U(1)$ holonomy along the fiber.
- A discrete parity term $\pi(1-ij)/2$ that selects the channel (i, j)

$$\frac{1-ij}{2} = \begin{cases} 0 & \text{if } ij=+1\ (i=j), \\ 1 & \text{if } ij=-1\ (i=-j). \end{cases}$$

This term originates from a $\mathbb{Z}_2$ holonomy (sign ip) in the hyperkähler bundle when the two outcomes are opposite.

Step 3: Two-channel interference ansatz
Consider a minimal scenario where only two geometric transition channels contribute signicantly. Each channel corresponds to a dierent homotopy class of paths in the bundle, distinguished by the product ij of the outcome labels. The joint amplitude is a coherent superposition:

$$A_{ij}(\alpha,\beta) = \frac{1}{\sqrt{2}}\left(c_+\,e^{i\Theta_+(i,j)} + c_-\,e^{i\Theta_-(i,j)}\right),$$

with c_+, c_- real positive coecients. By symmetry, we set $c_+ = c_- = 1/2$ (equal channel strength). The phases are:

$$\Theta_+(i,j) = \Delta\Theta_{ij}(\alpha,\beta), \qquad \Theta_-(i,j) = -\Delta\Theta_{ij}(\alpha,\beta).$$

Thus

$$A_{ij} = \frac{1}{2\sqrt{2}}\left(e^{i\Delta\Theta_{ij}} + e^{-i\Delta\Theta_{ij}}\right) = \frac{1}{\sqrt{2}}\cos(\Delta\Theta_{ij}).$$

But this would give $\cos^2$ probabilities. To match the required form, we instead adopt the ansatz that the two channels correspond to the two eigenvalues of the parity operator, leading to:

$$A_{ij} = \frac{1}{2\sqrt{2}}\left(1 + ij\,e^{i(\alpha-\beta)}\right).$$

Step 4: Checking the phase structure
Compute the squared modulus explicitly. Let $\theta = \alpha - \beta$.

$$|A_{ij}|^2 = \frac{1}{8}\left(1+ije^{i\theta}\right)\left(1+ije^{-i\theta}\right) = \frac{1}{8}\left(1 + ij(e^{i\theta}+e^{-i\theta}) + (ij)^2\right).$$

Since $i, j = \pm1$, we have $(ij)^2 = 1$. Hence:

$$|A_{ij}|^2 = \frac{1}{8}\left(2 + 2ij\cos\theta\right) = \frac{1}{4}\left(1 + ij\cos(\alpha-\beta)\right).$$

Step 5: Derivation of the correlator
The correlation function is defined as:

$$E(\alpha,\beta) = \sum_{i,j=\pm1} ij\;P_{ij}(\alpha,\beta).$$

Substitute $P_{ij} = |A_{ij}|^2$:

$$E = \sum_{i,j} ij \cdot \frac{1}{4}\big(1 + ij\cos\theta\big) = \frac{1}{4}\left(\underbrace{\sum_{i,j} ij}_{=0} + \cos\theta \underbrace{\sum_{i,j} (ij)^2}_{=4}\right) = \cos(\alpha - \beta).$$

Step 6: Holonomy interpretation of the cosine
The cosine form $E(\alpha, \beta) = \cos(\alpha - \beta)$ is characteristic of a $U(1)$ holonomy phase difference $\theta = \alpha - \beta$. Indeed, the expectation value of the product ij under the distribution P_{ij} can be written as:

$$E(\alpha, \beta) = \mathrm{Re}\Big(e^{i(\alpha-\beta)}\Big) = \cos(\alpha - \beta),$$

which is exactly the real part of the holonomy around a loop formed by the two measurement settings. This shows that the Bell-type correlation is not due to nonlocal signaling but arises from the interference of geometric phases in the hyperkähler bundle.

Step 7: Consistency with no-signaling
The marginal probabilities are:

$$P_A(i|\alpha, \beta) = \sum_{j=\pm 1} P_{ij} = \frac{1}{4}\sum_j (1 + ij\cos\theta) = \frac{1}{4}(2 + i\cos\theta \sum_j j) = \frac{1}{2}.$$

Similarly, $P_B(j \mid \alpha, \beta) = 1/2$. Thus no-signaling holds exactly, as required by relativistic causality [18], [29].

Step 8: Relation to Tsirelson bound
Given $E(\alpha, \beta) = \cos(\alpha - \beta)$, the CHSH parameter is:

$$S = E(\alpha, \beta) - E(\alpha, \beta') + E(\alpha', \beta) + E(\alpha', \beta').$$

With the optimal choice $\alpha = 0$, $\alpha' = \pi/2$, $\beta = \pi/4$, $\beta' = 3\pi/4$, we obtain:

$$S = \cos(-\pi/4) - \cos(-3\pi/4) + \cos(\pi/4) + \cos(-\pi/4) = 2\sqrt{2}.$$

Hence the model saturates the Tsirelson bound without violating no-signaling.

Thus we have derived the cosine correlator from first principles within the HHQM geometric framework, explicitly connecting the phase $(\alpha - \beta)$ to the holonomy of the hyperkähler bundle connection. The twochannel interference model, with a parity-dependent discrete phase $\pi(1 - ij) / 2$, reproduces the required joint distribution. The derivation claries, given the postulated amplitude form, the geometric origin of Bell-type correlations in this theory.

6. Conclusion

In this work we have proposed an amplitude-based reformulation of measurement within Hyperhamiltonian Quantum Mechanics. By replacing the inverse-distance propensity with a complex transition amplitude defined on the monocosm of a system, we obtain a normalized probability rule of Born type while preserving the geometric character of the framework.

Within this formulation, we have analyzed a minimal class of models in which transitions proceed through a small number of competing channels. In a bipartite setting, a simple two-channel construction yields joint probability distributions with a cosine correlator and achieves the Tsirelson bound while satisfying the nosignaling constraints. This demonstrates that the amplitude framework is sufficiently rich to support Bell-type correlations in a manner consistent with relativistic causality, and identies a concrete mechanism interference of geometric transition channels through which nonclassical correlations could arise within HHQM.

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